Turbulent Mixing of Coaxial Jets

Louis J. Alpinieri*
Polytechnic Institute of Brooklyn, Freeport, N. Y.

Experimental results on the turbulent mixing process between carbon dioxide and hydrogen central jets exhausting into a moving concentric stream of air are presented. The diffusion of mass and momentum only are considered, since both streams are at approximately equal temperature. The flow velocities are in the low to high subsonic range. Principal points of investigation are the condition of equal velocity between jets and the condition of equal mass flow per unit area between jets. Radial and axial distributions of concentration and velocity are presented. On the basis of the measurements, it is demonstrated that the product of local density and eddy kinematic viscosity coefficient can be assumed to be solely dependent upon the axial coordinate. Furthermore, it is concluded that no tendency toward segregation of the streams exists when either the velocities or the mass flows of the streams are equal. In agreement with previous results, it is found that mass appears to diffuse more readily than momentum. Finally, a formulation of the eddy viscosity and eddy diffusivity coefficients that adequately describe the results obtained is derived.

Nomenclature

a = radius of the central jet
b = length proportional to the width of the mixing region
$b_{1/2}$ = half-radius of the mixing region; for velocity defined as the value of r for which $u = u_e + u_c/2$: for concentration defined as the value of r for which $K = K_c/2$
I_0 = modified Bessel function of the first kind
k = constant
K = mass concentration of the central jet gas
P = static pressure
P_0 = stagnation pressure
r = radial coordinate
S_t = Schmidt number
T = static temperature
T_0 = stagnation temperature
u = axial component of velocity
v = radial component of velocity
x = axial coordinate
ϵ = eddy kinematic viscosity coefficient
ξ_K = transformed axial coordinate for concentration
ξ_u = transformed axial coordinate for velocity
ρ = density
τ_{turb} = turbulent shear stress
ψ = transformed radial coordinate

Subscripts

c = centerline property
e = property of the external stream
j = property of the jet
max = maximum value at a given axial position
min = minimum value at a given axial position

1. Introduction

THE turbulent mixing process between two coaxial jets has recently received an increasing amount of attention. The practical motivation for this has been the application to wakes and ramjet combustors. However, in addition, the problem affords an opportunity to gain a better understanding of the turbulent transport processes.

The problem under consideration is the mixing of an axisymmetric jet of other than air composition exhausting into a moving external stream at a sufficiently high Reynolds number to insure that the transport processes between streams will be turbulent. The most closely related studies are as follows.

Forstall and Shapiro[1] studied the diffusion of mass and momentum between coaxial jets of air where the central jet was composed of approximately 10% of helium for use as a tracer gas. A very extensive bibliography of the literature on turbulent jets is included with their results. The velocities considered in their experiments were in the low subsonic range. They concluded that mass diffusion was more rapid than momentum diffusion. In addition, the parameter that measured the relative rates of transfer of mass and momentum, the Schmidt number, was found to be independent of the velocity ratio between jets. However, in all cases, the central jet velocity was considerably larger than that of the outer jet since the maximum velocity ratio investigated was 0.5. A similar result regarding the relative rates of transfer of mass and momentum was obtained by Keagy and Weller[2] in experiments on helium and carbon dioxide jets exhausting into quiescent air.

The relative rates of diffusion of momentum and energy in turbulent jets were perhaps first studied by Ruden,[3]† who found that energy diffused more rapidly than momentum. A similar result was obtained by Alexander et al.,[4] who studied the decay of a ducted air jet which formed an external stream by entrainment. However, again, only the case where the central jet was of considerably larger velocity than the external stream was investigated.

The effect of large density differences were investigated by Corrsin and Uberoi[5] using a heated jet which exhausted into a quiescent region of different density. It was found that a decrease of jet density with respect to that of the receiving medium caused an increase in the rate of decay of the jet.

Although the analytical study of the turbulent decay of an incompressible jet is well documented by Schlichting,[6] relatively little appears regarding the more complex problem of a jet exhausting into a moving stream with large density gradients throughout the flow field. Recently, an approximate technique by Libby,[7] which yields solutions for the jet mixing problem with nonconstant density and with arbitrary

Received September 19, 1963; revision received June 19, 1964. This research has been supported by the Air Force Office of Scientific Research under Contract No. AF 49(638)-217 and the Aeronautical Research Laboratory, Office of Aerospace Research, under Contract No. AF 33(616)-7661. The work is based upon a dissertation submitted to the Polytechnic Institute of Brooklyn in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Astronautics). The author wishes to express his gratitude for the help and encouragement given him by Antonio Ferri, Paul A. Libby, and Victor Zakkay of the Polytechnic Institute of Brooklyn.

* Formerly Research Associate, Department of Aerospace Engineering and Applied Mechanics, Aerospace Institute; presently Member of the Technical Staff, Aerospace Corporation, El Segundo, Calif. Member AIAA.

† The author is indebted to the reviewer for pointing out this reference.

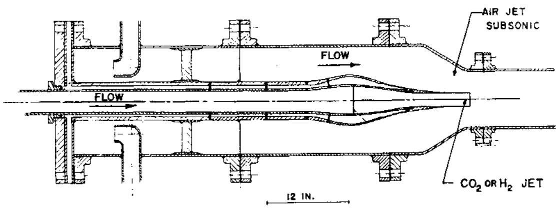

Fig. 1 Cross-sectional view of the wind tunnel.

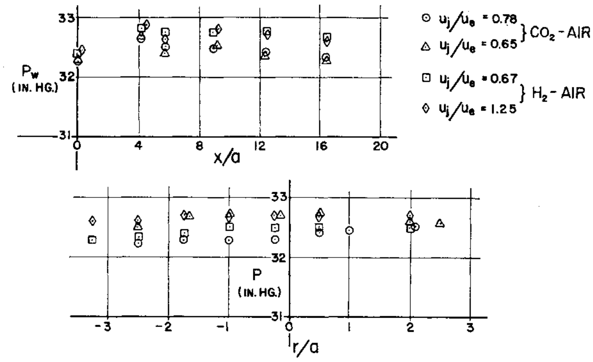

Fig. 2 Wall static pressure and radial distribution of static pressure at $x/a = 5.0$.

initial conditions, has appeared. Essentially, the method consists of a transformation and linearization of the equations of motion, mass diffusion, and energy, in order to render them into the form of the parabolic heat conduction equation. In order to obtain this form, the product ($\rho\epsilon$) must be assumed to be a function of the axial coordinate only. The solution of the problem in transformed coordinates is then straightforward. However, the transformation back to the physical plane requires the specification of the coefficients of transport of mass, momentum, and energy.

The greatest amount of attention has been focused on the formulation of the coefficient of momentum transport, or eddy viscosity. A knowledge of the eddy viscosity together with an empirical knowledge of the turbulent Schmidt and Prandtl numbers can then be used to determine the coefficients of mass and energy transport.

In order to correlate extensive experimental data due to Reichardt[8] on the decay of an incompressible jet, Prandtl[9] postulated a form for the eddy viscosity. This semiempirical formulation consisted of taking ϵ constant at any axial position and proportional to the maximum difference of the time-mean flow velocity. However, Reichardt's experiments consisted of a single jet exhausting into a stationary ambient. Thus, at every axial position, the minimum velocity was that of the ambient which was at rest. Therefore, no consideration was given to the case where the minimum velocity was nonzero. Since the eddy viscosity was taken proportional to a velocity difference, the condition of a jet exhausting into an ambient moving at the same velocity should produce nearly segregated streams. This condition is not reasonable, particularly when the jets are of different composition.

Some preliminary results that indicated that no minimum mixing or tendency toward jet segregation occurred when concentric jets are of equal velocity but unequal composition were reported by Ferri.[10] A proposed explanation for this anomalous behavior consisted of attributing the mixing to the fact that large gradients in density existed in the flow field and could alter the form of the eddy viscosity coefficient. This alteration was assumed to be that the product ($\rho\epsilon$) could be taken to be proportional to the difference $|\rho_e u_e - \rho_c u_c|$. Use of such a formulation indicated that the term ($\rho\epsilon$) was indeed a function of the axial coordinate only. However, the difficulty with this approach is that, for the special case when the ratio $\rho_c u_c/\rho_e u_e$ is initially equal to unity, a segregation of the jets is again predicted.

In order to gain a better understanding of the formulation of the turbulent transport coefficients, the present experimental investigation was performed. Essentially, the three main points of investigation are:

1) The validity of the assumption that the product of transport coefficient and local density is a function of the axial coordinate only.

2) The nature of the turbulent diffusion process between jets when the velocity ratio between them approaches unity.

3) The nature of the turbulent diffusion process between jets when the mass flow ratio between them approaches unity.

In order to verify the assumption of taking ($\rho\epsilon$) to be a function of x only, the analysis given in Ref. 10 is used to correlate the data. In order to test at a condition of equal velocities in the jet and external stream while still maintaining a large density difference, the decay of a jet of hydrogen exhausting into a moving stream of air was studied. The ratios of jet to external stream velocity ranged from less than to greater than unity. To achieve equal mass flow ratios in the jet and external stream and also to study the effect of jet density larger than that of the external stream, the decay of a carbon dioxide jet exhausting into a moving air stream was studied. The ratio of jet mass flow per unit area to external stream mass flow per unit area ranged from less than to greater than unity.

2. Apparatus, Test Conditions, and Data Reduction

Wind Tunnel

All tests were carried out by utilizing the high-pressure test facilities at the Aerospace Institute of the Polytechnic Institute of Brooklyn. In order to study the mixing of two coaxial jets, the wind tunnel shown in Fig. 1 was constructed. The central jet, consisting of either hydrogen or carbon dioxide, was introduced at the rear of the tunnel and was supplied from commercially available high-pressure bottles. The external stream was supplied by the laboratory's high-pressure air storage facilities. The diameter of the outer jet was 8 in. and the central jet diameter was 2 in.

The mass flow rate of the central jet was measured by recording the total and throat static pressure from a venturi-type flow meter located in the jet supply line. The mass flow of the external stream was obtained from measurements of the settling chamber total pressure and the static pressure in the plane of the mixing interface. Static pressures were measured along the test section wall and the radial distribution of static pressure was measured by a static pressure rake. These data are presented in Fig. 2. On the basis of these measurements, the static pressure throughout the mixing region was taken to be constant and equal to 32.5 in. Hg absolute.

In order to compute velocities from these measurements, the stagnation temperature of both jets was measured by iron-constantan-type thermocouples. The test conditions employed in the present experiment are summarized in Table 1.

Table 1 Test conditions

Jet gas	P_{0e}, in. Hg	T_{0e}, °R	T_{0j}, °R	$\frac{u_j}{u_e}$	$\frac{\rho_j u_j}{\rho_e u_e}$
H_2	42.5	484 ± 10	520 ± 10	0.66	0.0396
	42.5	484 ± 10	520 ± 10	0.95	0.0567
	42.5	484 ± 10	520 ± 10	1.25	0.0742
CO_2	42.5	484 ± 10	490 ± 10	0.47	0.66
	42.5	484 ± 10	490 ± 10	0.65	0.95
	42.5	484 ± 10	490 ± 10	0.78	1.17

For all tests, the velocity of the external stream was maintained at 650 fps.

Measuring Rakes

The mixing region was surveyed at various axial positions in order to obtain the radial distributions of velocity and concentration. In order to do so, measurements of static pressure, total pressure, and concentration were made. The static pressure rake used consisted of $\frac{1}{16}$-in.-diam stainless-steel tubes having a blunt nose and having peripheral orifices located fourteen probe diameters downstream of the nose.

The total pressure rake was used for both the measurement of total pressure and for the collection of gas samples. The nose shape of the probes was made by forming a $\frac{1}{16}$-in. tube into a 30° conical cavity such that the entrance diameter of the tube was reduced to 0.018 in. This entrance diameter then rapidly expanded to 0.042 in., which was the inside diameter of the tubing. Although the nose shape of the probe had little effect on the measurement of total pressure, it was found that improper probe design could radically effect the measurements of concentration.

The object of the internally expanded design was to minimize any probe induced disturbance of the flow field which might cause a distortion of the gas composition near the probe entrance. With flat faced constant inside diameter probes, very significant losses of the light gas component in the collected samples was noticed. This phenomena was most probably due to choking inside the probe which disturbed the flow upstream of the probe entrance. With the internally expanded probe, the phenomena was absent. The validity of the concentration measurements was determined by an integration of the jet mass flow contained in the measured radial profiles. This result was then compared with the mass flow given by the flow meter and in all cases checked within 20%.

Concentration Measurements

Gas samples were withdrawn from the mixing region through the total pressure tubes and collected in a series of vacuum bottles. At the conclusion of the test run, these samples were introduced to a thermal conductivity cell whose output was proportional to the concentration of jet gas in the sample. The proportionality between cell output and concentration was determined by a calibration which utilized gas samples of known composition. The calibration and construction of the gas sampling equipment are given in Ref. 11.

Data Reduction

Since the static pressure was taken constant throughout the flow field, the measurement of total pressure determined the local mean Mach number. The value of the specific heat ratio for the H_2-air mixtures was simply equal to 1.4. However, for the CO_2-air mixtures, its local value had to be computed from the local concentration measurements. No correction was made for the effect of density and velocity fluctuations on the total pressure measurements. An analysis of these effects is given in Ref. 5.

The mean Mach number and mean concentration data were plotted vs the radial coordinate for each axial position. Since the total temperature of both streams was nearly equal, its value was taken to be constant for the entire flow field. Thus, for various corresponding radial positions, the mean velocity profile could be computed from the Mach number and concentration data.

3. Theoretical Considerations

The governing equations for the turbulent mixing of two nonhomogeneous, nonreacting, isoenergetic axisymmetric jets in the absence of chemical reaction and radial and axial static pressure gradients are given in (7) and (10) as follows:

Conservation of Mass

$$\frac{\partial}{\partial x}(\rho u) + \frac{1}{r}\frac{\partial}{\partial r}(\rho v r) = 0 \tag{1}$$

Conservation of Momentum

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\rho \epsilon r \frac{\partial u}{\partial r}\right) \tag{2}$$

Conservation of Species

$$\rho u \frac{\partial K}{\partial x} + \rho v \frac{\partial K}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\rho \epsilon S_t^{-1} r \frac{\partial K}{\partial r}\right) \tag{3}$$

where the mean values of the independent variables are implied. Here the transport of momentum is considered as given by the assumption that, in analogy to laminar flow, the turbulent shear stress may be written

$$\tau_{\text{turb}} = \rho\epsilon(\partial u/\partial r) \tag{4}$$

where ϵ is a phenomenological quantity.

Libby[7] and Ferri et al.[10] have given a solution to the preceding system of equations which has been successfully applied to the correlation of extensive data on the decay of turbulent jets exhausting into a quiescent ambient. For brevity, only the most pertinent features of their analysis are presented here.

First, a stream function ψ is introduced to satisfy the conservation of mass equation as

$$\rho u r = \rho_j u_j \psi(\partial\psi/\partial r) \tag{5}$$

$$\rho v r = \rho_j u_j \psi(\partial\psi/\partial x) \tag{6}$$

A von Mises transformation is then applied such that the radial coordinate is replaced by the stream function. Next it is assumed that the products $(\rho\epsilon)$ and $(S_t^{-1}\rho\epsilon)$ may be replaced by their values on the axis (i.e., they are assumed independent of r). Finally, if the real axial coordinate is replaced by

$$\xi_u = \int_0^x \frac{(\rho\epsilon)_c}{\rho_j u_j a}\,dx \tag{7}$$

$$\xi_K = \int_0^x \frac{(S_t^{-1}\rho\epsilon)_c}{\rho_j u_j a}\,dx \tag{8}$$

the momentum and species equations are brought into the form of the parabolic heat conduction equation as

$$\frac{\partial u}{\partial \xi_u} = \frac{a}{\psi}\frac{\partial}{\partial\psi}\left(\psi\frac{\partial u}{\partial\psi}\right) \tag{9}$$

$$\frac{\partial K}{\partial \xi_K} = \frac{a}{\psi}\frac{\partial}{\partial\psi}\left(\psi\frac{\partial K}{\partial\psi}\right) \tag{10}$$

The boundary conditions for the problem of two coaxial jets of which the outer is semi-infinite are given by specifying the velocity and concentration completely along the line $\xi_K = \xi_u = 0$ and enforcing that as $\psi \to \infty$, $u/u_j \to u_e/u_j$ and $K \to 0$. Under these conditions, the solutions to Eqs. (9) and (10) are given in Ref. 10 as

$$\frac{u}{u_j}(\psi, \xi_u) = \frac{a}{2\xi_u}\exp\frac{-(\psi/a)^2}{4(\xi_u/a)} \times \int_0^\infty \exp\frac{-(\psi'/a)^2}{4\xi_u/a} I_0\left(\frac{\psi\psi'}{2\xi_u a}\right) f(\psi')\psi' d\psi' + \frac{u_e}{u_j} \tag{11}$$

and

$$K(\psi, \xi_K) = \frac{a}{2\xi_K}\exp\frac{-(\psi/a)^2}{4(\xi_K/a)} \times \int_0^\infty \exp\frac{-(\psi'/a)^2}{4\xi_K/a} I_0\left(\frac{\psi\psi'}{2\xi_K a}\right) g(\psi')\psi' d\psi' \tag{12}$$

where

$$f(\psi') = (u/u_j - u_e/u_j)_{\xi_u=0}$$
$$g(\psi') = (K/K_j)_{\xi_K=0}$$

are the initial profiles of velocity and concentration, respectively.

Up to this point, the obvious simplification of assuming a step-profile in velocity and concentration as an initial condition has been avoided. The reason for this is that, for any jet experiment, a step-profile, at least in velocity, is an initial condition that is impossible to achieve. This is because of the inevitable accumulation of boundary layer on the walls of the jets.

An important observation that can be made from the above analysis is that, as a consequence of the assumed independence of the transport coefficients on the radial coordinate, the radial distribution of the flow quantities, at any downstream axial position where the centerline values have been experimentally obtained, can be determined independently of the actual value of the transport coefficient. This fact will be used in the analysis of the experimental data in order to verify the assumption of taking $(\rho\epsilon)_c$ and $(\rho\epsilon S_t^{-1})$ independent of the radial coordinate.

4. Analysis of the Experimental Data

The results of the previous analysis indicate that a considerable simplification of the governing equations can be achieved by assuming that the transport coefficients can be taken to be independent of the radial coordinate. If this assumption can be verified, then the effects of the transport coefficients are confined to the prediction of the centerline decay of concentration and velocity only. Alternatively, if the problem is viewed in reverse, then if there is no a priori knowledge of the transport coefficients, an experimental determination of the centerline decays will yield at least the numerical values of these quantities.

In order to verify the assumptions that $(\rho\epsilon)$ and $(S_t^{-1}\rho\epsilon)$ are solely functions of x, use is made of the fact that, for given initial conditions, a specification of the centerline values of velocity and concentration at any other downstream position will completely determine the associated radial distribution of velocity and concentration. Then, if an agreement between the analytical and the experimental radial distributions exists, it would indicate that the transport coefficients are indeed independent of the radial coordinate. However, before proceeding with the analysis, attention must be given to selecting the initial conditions for the calculation.

In any real experimental situation, there is an inevitable accumulation of boundary layer on the jet dividing boundary. Because this boundary layer is usually quite small, it is particularly difficult to measure its properties. Furthermore, theoretical calculations given in Ref. 10 indicate that even very small boundary layers can significantly effect the length of the potential core and the resulting downstream decays. In order to avoid making an error in initial condition data by assuming either a step profile or a computed boundary layer, the calculation of the mixing region was started at a position downstream of the mixing interface where measured data were available. These data were then used as initial conditions and were selected at a location where the concentration and total pressure measurements indicated a vanished potential core, and, in addition, showed no defects, which would indicate that the momentum defect of the boundary layers had not been eliminated. The axial stations at which the measurements satisfied these conditions were $x/a = 5.0$ for the hydrogen tests and $x/a = 10.5$ for the carbon dioxide tests. The measured initial value data for the hydrogen mixing computation are shown in Fig. 3, whereas the measured initial value data for the carbon dioxide mixing are shown in Fig. 4. After performing the inversion into ψ coordinates by the use of Eq. (5), the resulting initial profiles $f(\psi') = [u/u_j(\psi, 0) - u_e/u_j]$ and $g(\psi') = K/K_j(\psi, 0)$ were used to compute the centerline decay of concentration and velocity in the transformed plane given by Eqs. (11) and (12). However, the transformation back to the physical plane given by Eqs. (7) and (8) required a knowledge of the transport coefficients $(\rho\epsilon)_c$ and $(S_t^{-1}\rho\epsilon)_c$. Essentially, the transport coefficients can be considered to be the coordinate stretchers that will fit a result in the ξ plane to a result in the x plane. Therefore, an alternative procedure to a knowledge of the value of the transport coefficient is a knowledge of the centerline decays in terms of the physical coordinates. From the measurements these data were available, and thus by use of the measured centerline values of velocity and concentration, together with the computed decay of u_c/u_j vs ξ_u and K_c vs ξ_K, a point by point correlation of ξ_u to x and ξ_K to x was obtained. These data are shown in Figs. 5 and 6. In addition, it can be seen by considering Eqs. (7) and (8) that the value of the Schmidt number is given by

$$S_t = (d\xi_u/dx)/(d\xi_K/dx) \tag{13}$$

The values of S_t so obtained varied between 0.5 and 0.7 for the carbon dioxide tests which was in reasonably good agreement with the results obtained and quoted in Ref. 1. The range in Schmidt number obtained in these tests could

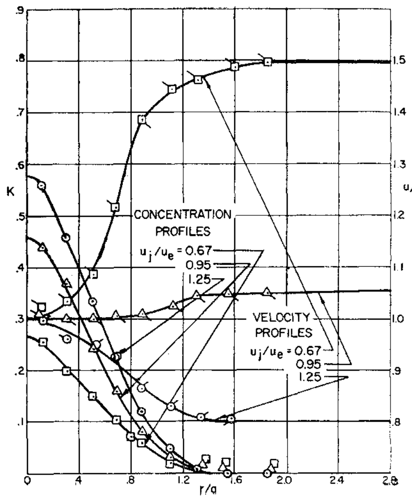

Fig. 3 Initial profiles of concentration and velocity at $x/a = 5.0$, hydrogen-air mixing.

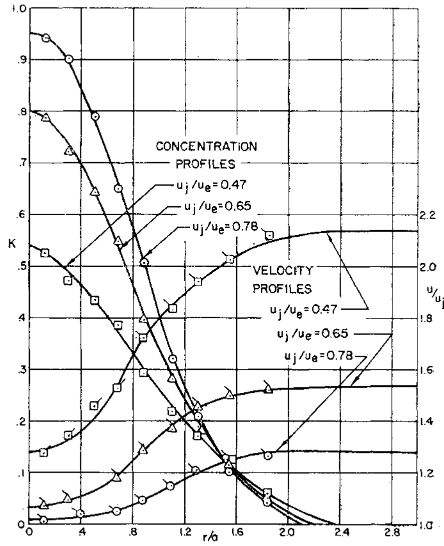

Fig. 4 Initial profiles of concentraion and velocity at $x/a = 10.5$, carbon dioxide-air mixing.

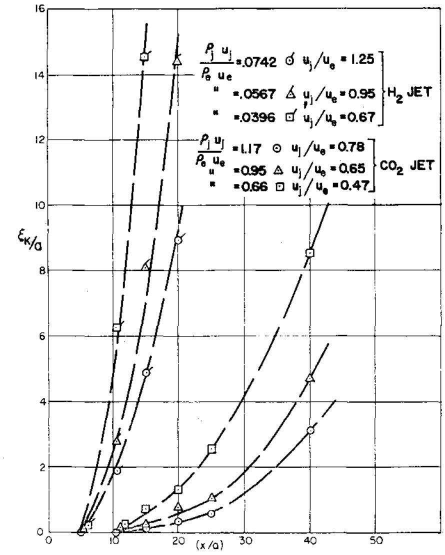

Fig. 5 Relation between the transformed and physical coordinates for concentration.

be attributed to the method of calculation rather than to any physical variation of this quantity. The reason for this can be explained as follows: In considering a plot of u_c/u_j vs ξ_u, it becomes readily apparent that, owing to the relatively small velocity differences between jet and external stream, for these experiments, the value of u_c/u_j very quickly becomes asymptotic to the value u_e/u_j. Thus, because the derivative $d(u_c/u_j)/d\xi u$ becomes small very quickly, a small error in the measured value of u_c/u_j can produce a very significant error in the value of ξ_u so obtained. Although the concentration measurements were assumed to be of better precision than those of velocity, the same argument would apply in determining the value of ξ_K, though perhaps to a lesser extent. However, a consequence of the fact that, in the downstream region, the centerline velocity rapidly becomes insensitive to a significant change in ξ_u indicates that the effect of Schmidt number is small. This conclusion is evidenced by reference to Eqs. (5) and (13). It can be seen that the effect of a nonunity Schmidt number is to make ξ_u different from ξ_K for a given value of x. If one considers that for a fixed value of K_c (fixed ξ_K) a large variation of ξ_u (implying a large variation of S_t) produces a small variation in u_c/u_j, it follows that only a small change in the relation between r and ψ as given by Eq. (5) will result. As the calculation proceeds downstream and the derivative $d(u_c/u_j)/d\xi u$ becomes smaller, the relation between r and ψ becomes all the more insensitive to a change in Schmidt number.

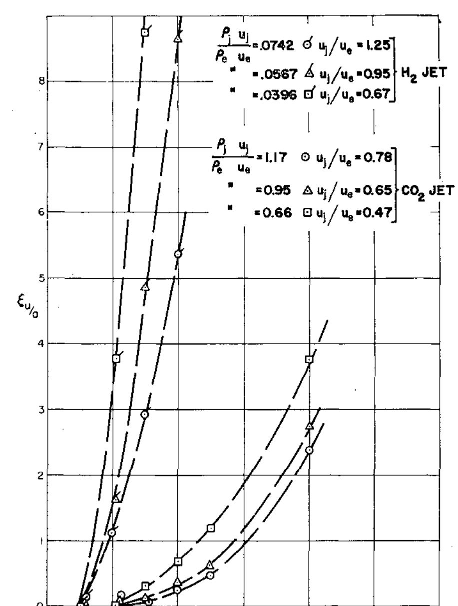

Fig. 6 Relation between the transformed and physical coordinates for velocity.

In the present calculation of the downstream radial profiles, the observed values of S_t for the carbon dioxide tests were used. However, since the velocity differences for the hydrogen tests were small, an axial velocity decay sufficiently precise to sensibly determine the Schmidt number was not obtained. However, in light of the argument regarding the diminishing influence of Schmidt number in the downstream direction, it was found that any average value of S_t could not significantly affect the resulting downstream radial concentration distributions. To illustrate this, values of $S_t = 0.6$ and $S_t = 1.0$ were arbitrarily selected for the computation of the radial distributions of hydrogen concentration.

Once the relation between the physical and transformed coordinates was obtained, the radial distributions of concentration and velocity proceeded straightforward from Eqs. (11) and (12). The solutions obtained $K(\psi, \xi_K)$ and $u/u_j(\psi, \xi_u)$ or, alternatively, $K(\psi, x)$ and $u/u_j(\psi, x)$ were then inverted to the physical radial coordinate by use of Eq. (5). The results of the calculation for the radial concentration distribution of hydrogen and carbon dioxide tests are shown in Figs. 7 and 8, respectively. In addition, the correlation with the measurements of the radial distribution of velocity for the carbon dioxide tests is shown in Fig. 9. The radial distribution of velocity for the hydrogen tests is not presented, since, in this case, the calculation of velocity is so critically dependent on the precision of the concentration measurements that a small error in the measurement of concentration could produce an error in velocity of the order of the total initial velocity difference between the jet and external stream. The insensitivity of the radial distributions to errors in the value of S_t is shown in Fig. 8 where it can be seen that virtually no difference exists for $S_t = 1.0$ or $S_t = 0.6$.

The agreement shown for the measurements presented can certainly be considered satisfactory. It must again be noted that as yet no statement has been made regarding the kinematic viscosity or diffusivity other than that these quantities are solely functions of the streamwise coordinate. Thus, the agreement shown certainly justifies this assumption, since, if the transport coefficients varied significantly with the radial coordinate, we could not expect the results obtained. Therefore, it can safely be said that the eddy kinematic viscosity and diffusivity are solely functions of x, and their values are intimately related to the data presented which relate the transformed coordinates ξ_u and ξ_K to the physical coordinate x through the expressions (7) and (8).

5. Discussion of the Eddy Kinematic Viscosity

The essential difference between the treatment of laminar and turbulent flows lies in the formulation of an expression for the transfer mechanism of the turbulent process. In laminar flow, the shear stress is given by the product of viscosity coefficient and velocity gradient. However, in turbulent flow, no such simple relation exists. Nevertheless, the usual procedure has been to relate ϵ to the local gradients or at least the mean values of the local gradients of the flow properties of the gas. Although ϵ is in general a function of all the space coordinates, it appears that the product $\rho\epsilon$ can be taken to be solely a function of the axial coordinate. However, the axial variation of the eddy viscosity for the case presently under consideration is not well established.

Several formulations of the eddy viscosity coefficient as applied to various types of problems are summarized in Ref. 6. These results are derived from a combination of semi-

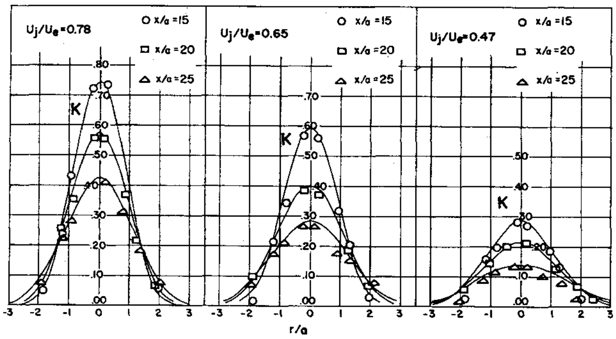

Fig. 7 Radial distribution of concentration of u_j/u_e = 0.78, 0.65, and 0.47 in carbon dioxide-air mixing.

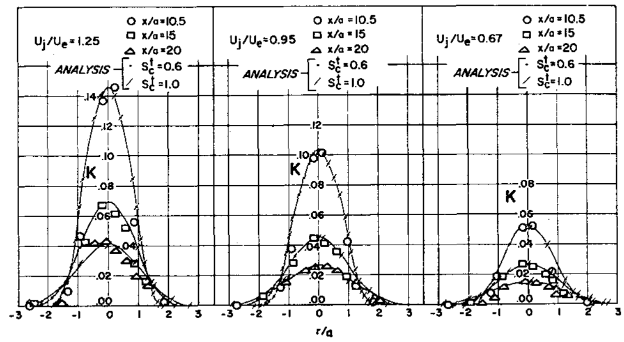

Fig. 8 Radial distribution of concentration for u_j/u_e = 1.25, 0.95, and 0.67 in hydrogen-air mixing.

empirical results, which have been related to the Prandtl mixing length theory. The classical case, most applicable to the present experiments, is that of a circular jet exhausting into a stationary stream for which Prandtl introduced the following expression for the kinematic viscosity coefficient:

$$\epsilon = kb|u_{\max} - u_{\min}| \tag{14}$$

Here k is an empirical constant and b is proportional to the width of the mixing region. However, the measurements from which this formula developed dealt with the case $u_{\min} = 0$, and therefore the necessity for the inclusion of the term $u_{\min}$ had not been experimentally demonstrated. The presence of the term $u_{\min}$ in Eq. (14) necessarily gives rise to the question of what happens when $u_{\max}$ is made equal to $u_{\min}$, as for the case of equal velocity tangential jets. Then, if a region of turbulence separates the jets (e.g., the boundary layers on the walls of the jets at the origin of mixing), the Prandtl expression would predict a damping of this turbulence and an almost complete segregation of the jets. The experiments to be described contradict this result.

For the case of an incompressible jet exhausting into a quiescent region, the form of the eddy viscosity has been well established as

$$\epsilon = kb_{1/2}u_c \tag{15}$$

Here, the constant k has been found to be equal to 0.025 when $b_{1/2}$ is the half-radius defined as the radial distance to the point where the local velocity is one-half of its centerline value.

For an incompressible single circular jet, it has been shown that, asymptotically, the half radius is linearly proportional to x, whereas the centerline velocity is inversely proportional to the streamwise coordinate. Use of Eq. (15) then results in the eddy kinematic viscosity coefficient becoming a constant of the entire flow field in the far downstream region.

In the following discussion, the effects of variable density across the mixing region and the effects of a moving ambient on the formulation given by Eq. (15) will be investigated.

Effect of a Density Difference between the Jet and Receiving Medium

Experimental studies of a jet exhausting into a quiescent receiving medium of unequal density were performed in Ref. 5 by using heated jets, and in Ref. 2 by the use of jets of different composition. The asymptotic decay of velocity fit an x^{-1} power law reasonably well, whereas measurements of the half-radius presented in Ref. 5 indicated that the linearity of the half-radius with the axial coordinate observed in incompressible flow[8] was still valid. However, the rate of linear spread was found to be strongly dependent upon the difference in density between the jet and the receiving medium. Indeed, one conclusion reached in Ref. 5 is that a decrease in jet density with respect to that of the receiving medium caused an increase in the spreading rate of the jet.

Using the analysis presented in Sec. 3, an extensive correlation of all available single jet decay data was presented in Ref. 10. It can be shown that a consequence of this analysis is that for step-profile initial conditions, the centerline solution for the velocity in the ξ plane is independent of the actual value of the initial conditions. Also, the transformation of this solution back to the physical plane is completely determined by a term of the form $(\rho\epsilon)_c/\rho_j u_j a$ which appears in Eq. (7). Therefore, this term completely determines the behavior of the axial decay. It was found in Ref. 10 that, by taking

$$(\rho\epsilon)_c/\rho_j u_j a = 0.025(\rho_e/\rho_j)^{1/2} \tag{16}$$

in the fully developed mixing region, good correlation was obtained. It can be seen that Eq. (16) represents $(\rho\epsilon)_c$ to be a constant for the entire fully developed region which is consistent with the asymptotic result for incompressible flow. Furthermore, a result quoted in Ref. 10 indicated that the asymptotic solutions obtained from Eqs. (11) and (12), when step-profile initial conditions were assumed, could be used to a good degree of approximation even in regions approaching the end of the potential core. If the asymptotic solutions for half-radius and centerline velocity[10]

$$\lim_{\xi\to\infty} b_{1/2} = (\rho_j/\rho_e)^{1/2}\xi \qquad \lim_{\xi\to\infty} u_c = u_j a/\xi$$

are substituted in Eq. (16), the result becomes

$$(\rho\epsilon)_c = 0.025\rho_e b_{1/2} u_c \tag{17}$$

which, with the exception of the density term, is identical to the result obtained for the incompressible or constant density case. Hence, the effect of density variations in the flow field on the form of the eddy viscosity can be accounted for by the inclusion of a density term in the form given by Eq. (17). If Eqs. (16) and (17) are taken to be equivalent, which is certainly true in the downstream region, then again the product $b_{1/2}u_c$ must be a constant, which is consistent with the results of the measurements previously cited.

Effect of a Moving External Stream

Experimental studies of the decay of a jet exhausting into a moving coaxial stream were performed in Ref. 1 by using streams of unequal compositions, and in Ref. 2 by the use of streams of unequal temperature. However, in both cases

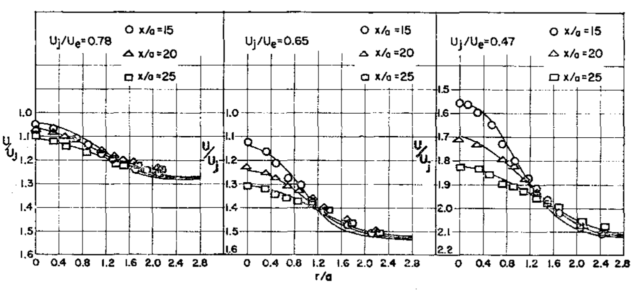

Fig. 9 Radial distribution of velocity for u_j/u_e = 0.78, 0.65, and 0.47 in carbon dioxide-air mixing.

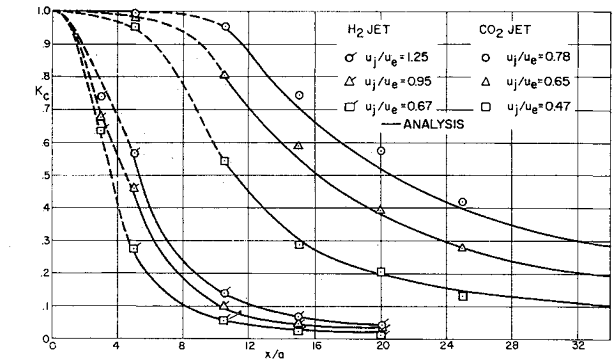

Fig. 10 Decay of centerline concentration.

the density variations were small enough for the experiment to be considered of a constant density type. In Ref. 1 it was noted that the presence of a moving external stream caused a decrease in the rate of spread of the central jet and that the rate of growth of half-radius was no longer linear. Furthermore it was found that $(u_c - u_e/u_j - u_e) \propto x^{-1}$. Thus, regardless of whether Eqs. (14) or (15) are taken as the formulation of the eddy viscosity coefficient, it becomes evident that ϵ must depend on the axial coordinate. Additional evidence for this statement is furnished by some recent experiments by Ragsdale and Weinstein[13] for a Bromine jet exhausting into a moving airstream. In the associated analysis, the eddy kinematic viscosity was assumed constant with respect to x. However, reference to the resulting correlation indicates that the value of ϵ chosen was probably a mean value with respect to the actual variation with the axial coordinate.

Eddy Kinematic Diffusivity

In Ref. 1, the value of the turbulent Schmidt number obtained was approximately 0.7, which is consistent with the results found in the present tests. Furthermore, the values of the half-radius for velocity and concentration found in Ref. 1 and also in the present investigation indicate that, to a good degree of approximation, the turbulent Schmidt number can be taken to be the ratio of the local values of half-radius of velocity to the half-radius of concentration. This approximation becomes exact in the downstream region where the ratio of the half-width of the concentration profile to that of velocity is exactly equal to the Schmidt number as expressed by Eq. (13). Thus, by specifying the form of the eddy viscosity in terms of the velocity half-radius, the eddy diffusivity can be obtained by a substitution of the concentration half-radius.

Correlation with the Present Experiments

The data obtained for the decay of centerline concentration and centerline velocity are presented in Figs. 10 and 11. It can be seen that, in agreement with the results previously quoted for the single jet, the lower density hydrogen jet decays much more rapidly than the higher density carbon dioxide jet.

According to Eq. (14), for the condition $u_{max} = u_{min}$, turbulent transfer should vanish and a minimum amount of mixing or tendency toward stream segregation should exist. Reference to the H_2 axial concentration data given in Fig. 10 indicates that no such minimum was found to exist. This result is further substantiated by some recent experiments by Zakkay and Krause[12] where it was found that increasing u_j/u_e through a value of unity solely produced a delay in the decay of the centerline properties.

In Ref. 10, an alternate form of the eddy viscosity for the case where large density gradients exist in the flow field was proposed as

$$\frac{(\rho\epsilon)_c}{\rho_j u_j a} = 0.025 \frac{b_{1/2}}{a} \frac{\rho_e u_e}{\rho_j u_j} \left| 1 - \frac{\rho_c u_c}{\rho_e u_e} \right| \tag{18}$$

This formulation was used with a good degree of success in Ref. 10 for the correlation of single jet decay data. However, a minimum of mixing is predicted when $\rho_j u_j/\rho_e u_e$ is made to approach unity. Testing through this condition was achieved in the CO_2 tests, and, as can be noted from the results appearing in Fig. 10, no such minimum was found to exist.

On the basis of these results, it seems reasonable to assume that no combination of jet and external stream flow properties will produce a vanishing eddy kinematic viscosity coefficient. It appears, however, that an increase of jet velocity or mass flow will simply delay the rate of decay of centerline properties.‡

In order to arrive at a possible formulation of the eddy viscosity, an extensive study of the centerline decay data was made. Consider now the plot of ξ_u vs x and ξ_K vs x given in Figs. 5 and 6. It can be seen that an essentially quadratic variation between ξ and x exists. Reference to Eqs. (7) and (8) then indicates that the functional form of the factors $(\rho\epsilon)_c/\rho_j u_j a$ and $(S_t^{-1}\rho\epsilon)_c/\rho_j u_j a$ should be linear with x. A plot of the half-radius vs x obtained for these experiments indicates that, in the range considered, the axial variation of this quantity is approximately linear. Therefore, the axial variation of the eddy viscosity can be obtained by incorporating the half-radius in its formulation.

On the basis of the preceding considerations, it was found that a possible formulation for the eddy viscosity which predicted the observed results was

$$\frac{(\rho\epsilon)_c}{\rho_j u_j a} = 0.025 \frac{b_{1/2}}{a} \left(\frac{\rho_e u_c}{\rho_j u_j} + \frac{\rho_e u_e^2}{\rho_j u_j^2} \right) \tag{19}$$

The results of a computation using this formulation are shown by the solid curves in Figs. 10 and 11. It can be seen that the agreement obtained is quite good. Furthermore, the expression degenerates to the single jet decay formulation, Eq. (17), when $u_e = 0$ and gives the classical result Eq. (15) for the case of uniform density and zero external stream velocity.

Reference to Fig. 10 shows that, as the velocity ratio u_j/u_e increases, the concentration at any given axial position must increase. Then, in the case of zero external stream velocity, we must have a higher centerline concentration than in the case of a nonzero external stream velocity. Therefore, the effect of a moving external stream is to increase the rate of

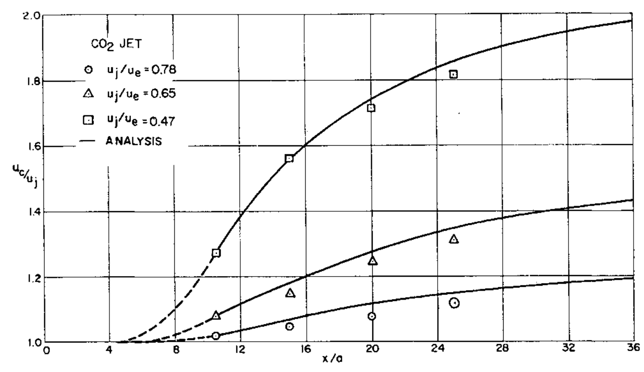

Fig. 11 Decay of centerline velocity.

‡ The reviewer has pointed out that since no tendency toward segregation of streams for the initial conditions herein reported was detected one is led to inquire about the results of an experiment where the jets are initially of equal momentum flux, i.e., $\rho_e u_e^2 = \rho_j u_j^2$. Unpublished experimental results through a unity value of momentum flux ratio have recently been performed and communicated to the author by V. Zakkay of the Polytechnic Institute of Brooklyn. With an argon jet exhausting into a moving stream of air, no tendency toward jet segregation was found.

entrainment of air by the jet. Then the larger the velocity of the external stream with respect to that of the jet, the larger will be the amount of air entrained. The reason is, perhaps, that any turbulent fluctuation present in the receiving medium is a fluctuation that would not exist if the receiving medium was at rest. In the case where the external stream is of greater density than the jet, the situation is particularly acute, since a small amount of air entrainment can rapidly cause a large change in the centerline concentration of jet gas. In the case where the jet gas is heavier than the receiving medium, the situation is not nearly as acute, since a larger amount of air entrainment is required in order to change the centerline concentration. This is evidenced by the much slower decay of the carbon dioxide jet that was observed.

If the Reynolds number is sufficiently large, then, regardless of the difference in the mean velocity between the two jets, turbulent fluctuations in the transverse velocity component will exist in both jet and external stream. Thus, although gradients of the mean velocity must contribute to the production of turbulence, they do not represent this production completely. It is entirely possible for a turbulent mechanism to exist in a decaying jet regardless of its initial condition. As a result, it should not be surprising that when two jets meet tangentially, turbulent transport can occur regardless of their relative mean-value initial conditions.

An intuitive approach to the coaxial jet problem is to con sider the jets separately as annular and axial type ejectors. In this case, the amount of the central jet that is entrained by the annular external stream must certainly depend upon the mass flow $\rho_e u_e$ and momentum $\rho_e u_e^2$. Similarly, the amount of external stream that can be entrained by the central jet must depend upon the jet mass flow $\rho_j u_j$ and momentum $\rho_j u_j^2$. Thus, it seems reasonable that the eddy viscosity, which is a measure of the momentum exchange, must depend upon the mass flow and momentum of both jets. Although these considerations cannot lead directly to the formulation given by Eq. (19), they do suggest the validity of the terms that are included. Nevertheless, verification of Eq. (19) for conditions of different velocity and mass ratio must await further experimental results.

Finally, it is worth noting that by following the analytical procedure herein reported it is possible to completely isolate the axisymmetric mixing effects from any extraneous considerations such as the initial boundary layers. That is, by beginning the mixing analysis at some point downstream of the actual origin of mixing, a regular measured profile of the flow properties can be used as an accurate initial condition. In this way both the theoretical and the experimental results for downstream profiles can be singularly attributed to the turbulent mixing process.

6. Conclusions

The results of an experimental investigation on the turbulent mixing of nonhomogeneous coaxial jets have been presented. On the basis of the measurements and the correlations obtained, the following conclusions may be made:

1) In the mixing region between jets of unequal density, the transport properties $(\rho\epsilon)_c$ and $(S_t^{-1}\rho\epsilon)_c$ can be considered to be solely functions of the axial coordinate.

2) No tendency toward segregation of the two jets is evident when either the velocity ratio, mass flow ratio, or momentum flux ratio is made equal to unity.

3) For the range of variables considered in this experiment, a possible formulation of the eddy viscosity coefficient is given by Eq. (19).

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